A Study on the Determination of Closing Level for Finite Element Analysis of Fatigue Crack Closure

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An elastic-plastic finite element analysis is performed to investigate detailed closure behavior of fatigue cracks and the numerical results are compared with experimental results. The finite element analysis performed under plane stress using 4-node isoparametric elements can predict fatigue crack closure behavior. The mesh of constant element size along crack surface can not predict the opening level of fatigue crack. The crack opening level for the constant mesh size increases linearly from initial crack growth. The crack opening level for variable mesh size, is almost flat after crack tip has passed the monotonic plastic zone. The prediction of crack opening level using the variable mesh size proportioning the reversed plastic zone size with the opening stress intensity factors presents a good agreement with the experimental data regardless of stress ratios.

Key Words : Fatigue Crack Growth, Finite Element Analysis, Crack Closure Behavior, Forward Plastic Zone Size, Reversed Plastic Zone Size, Prediction of Opening Point

1. Introduction

The precise evaluation of fatigue crack growth behavior is very important for life assessment of structures under cyclic loadings (Fuchs and Stephens, 1980, Kwon, 1990). Specially, the studies on crack closure behavior occurring during fatigue crack growth cannot be overlooked and have been widely performed by many researchers (Elber; 1970 and 1971).

The determination of fatigue crack closing level has been mostly measured by experimental method, which not only needs to spend time and high cost, but also is very difficult to apply to real structures. When experimental methods cannot be applied, the finite element method has long been used and has been provided useful results on fatigue crack growth and closure behavior under variable loadings (Ohji et al., 1974, Newman, (1976) and (1977)). This method has been carried out by many researchers because it has the merits of facilitating the changes of specimen geometries and material properties, loading histories and other factors. But the computational cost of finite element method for fatigue crack closure behavior is high. It is difficult to predict whether the analysis is precisely solved or not without accomplishing studies of susceptibility and accuracy for finite element model in case of complicated problem. Even though many researchers have provided useful information on fatigue crack closure behavior by finite element method, there are few study which made comparisons between finite element method and experimental results. Therefore, the finite element method to precisely predict the behavior of fatigue crack closure is essential. Recently the crack closure behavior at the residual stress fields was successfully studied (Choi and Song, 1995) and special attention was given to the determination of the most appropriate mesh size that would provide good numerical results (Park et al, 1997).

Park et al., (1997) reported that a unique and most- appropriate mesh size exists for a given loading condition that would provide good

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numerical results which were comparable with experimental data. And also they postulated that the ratio of the most appropriate mesh size to the theoretical monotonic plastic zone size usually decreased with an increase of maximum stress intensity factor K_{max} and the one in term of the theoretical reversed plastic zone size was nearly constant for the range of stress ratio R from 0 to 0.3. Through their study, they suggested the precise simulation method for fatigue crack closure behavior.

In this study, an elastic-plastic finite element analysis through modifying and improving Park et al. (1997)'s method is carried out to investigate detailed crack closure behavior. The effect on the relationship between plastic zone size and crack opening behavior, specially opening level, which was overlooked by Park et al. (1997)'s study will be evaluated. Finally, the advanced prediction method of crack opening level will be proposed.

2. Finite Element Model for Analysis of Crack Closure

The finite element code used for fatigue crack closure analysis in this study is developed by modifying a general elastic-plastic finite element code by Owen and Fawkes (1983). Small deformation theory is used in this code. The non-linear equations in the finite element analysis are solved using the direct Newton-Raphson method based on the tangent stiffness matrix. The theory of incremental rate-independent classical plasticity and the Von Mises criterion are chosen. Combined hardening (= (1/2) kinematic hardening + (1/2) isotropic hardening) is employed to sufficiently simulate the Bauschinger effect associated with reserved yield. The stress-strain relationship is modelled as a bilinear law. The constitutive equation is numerically integrated according to a mean normal method with subincrementation equation and radial return.

The opening and closing states of the crack surface are identified through monitoring all nodal displacement and nodal reaction forces along the crack at each load increment and then the boundary conditions are changed. If the vertical nodal displacement becomes zero, the crack surface is defined as closed and then the corresponding node is fixed as a roller support. If the nodal reaction force become zero, the crack surface is defined as open and then the corresponding node is released. Each load increment in this study corresponds to $(1/1000)\,\sigma_y$ where σ_y is the yield stress of material. The crack tip opening and closing load levels are defined as the load levels at which the nearest node behind the crack tip starts to open or to close, respectively. The computation is carried out on a IBM-SP2 supercomputer.

A comparison between numerical and experimental results is very important to derive quantitative conclusions from finite element method. The experimental crack opening data used for the comparison is the results obtained form Kim (1993)'s experiment report. Material properties used for the analysis are follows: Young's modulus E = 70 GPa, Poisson's ratio $\nu = 0.33$, yield stress $\sigma_y = 379$MPa, and the plastic modulus or the linear strain hardening parameter $H' = (d\sigma_y/d\varepsilon) = 0.01E = 700$MPa. These properties are expected to correspond to the 2024-T351 aluminium alloy which was used in the above mentioned work (Kim, 1993). A center-cracked tension (CCT), of initial notch $2a_0 = 16$mm, 10mm thick, 184mm long and 70mm wide, is used as a finite element model for numerical analysis. Four-node isoparametric elements are used. Only one-quarter of the specimen is modelled. Figure 1 shows the typical fine mesh configuration. In this study, the sizes of elements in the fine mesh zone along the crack line linearly increase. This will be explained later.

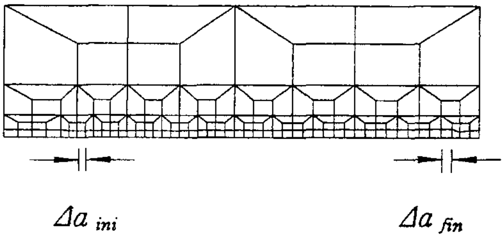


Fig. 1 Typical increasing fine mesh along crack line used in this study

3. Numerical Analysis

Before studying the determination method of crack closing level, the changes of the forward and reversed plastic zone sizes at the crack tip along crack growth will be analyzed.

3.1 Plastic zone of fatigue crack

Based on the studies of Choi and Song (1995), Park et al. (1997) and McClung and Sehitoglu (1989), the crack growth phenomenon through the numerical analysis is simulated as releasing a node along crack line when a nodal reaction force change from tension to compression force. The valid crack opening level is obtained when it becomes stable after the crack tip advances beyond the monotonic plastic zone induced by the initial crack length.

The monotonic plastic zone, the region of material experiencing plastic deformation when the cracked member is subject to the maximum load in the cycle, is defined for plane stress conditions as follows (McClung, 1991)

$$r_p=\frac{1}{\pi}\left(\frac{K_{max}}{\sigma_y}\right)^2 \quad (1)$$

The standard estimate for the size of the reversed (or cyclic) plastic zone size is also due to Rice (1967). It was suggested that the change in stresses, strains, and displacements due to load reversal were given by a solution identical to that for original monotonic loading, but with the loading parameter L replaced by the load reversal ΔL and the yield stress replaced by twice its value due to the doubling of the stress-strain curve for cyclic plasticity. The changes due to load reversal may be subtracted from the distributions corresponding to the original monotonic loading by the principle of superposition to determine the final distribution at minimum load. For zero-max loading, the size of the reversed plastic zone may be written

$$\Delta r_p=\frac{1}{\pi}\left(\frac{\Delta K}{2\sigma_y}\right)^2=\frac{1}{4\pi}\left(\frac{K_{max}}{\sigma_y}\right)^2=\frac{1}{4}r_p \quad (2)$$

where ΔK is stress intensity factor ranges.

It can be predicted that the ratio of reversed to forward plastic zone widths for zero-max loading is 1/4. More precisely, the yield strength σ_y should be replaced by the cyclic yield strength σ_{yc}, so that the size of the reversed plastic zone size is influenced by cyclic hardening or softening. For simplicity, this subtlety will be neglected in the current analysis. It is clear that the simple rule-of-thumb for reversed plastic zone sizes proposed by Rice, while roughly correct for stationary cracks, is not accurate for fatigue cracks which experience closure.

McClung (1991) developed a simple method to predict the size of the reversed plastic zone in fatigue which takes crack closure into account. The basic concept is a simple one. Reversed plastic flow at the crack tip is driven by a singular stress field as long as the crack tip is open. When the crack tip closes, the singularity vanishes, but some further reversed plasticity can occur due to nominal applied stresses. The size of the reversed plastic zone at the moment of first crack tip closure is given by

$$\Delta r_p=\frac{1}{\pi}\left(\frac{\Delta K_{eff}}{2\sigma_y}\right)^2 \quad (3)$$

where effective stress intensity factor ranges $\Delta K_{eff}=\Delta S_{eff}\sqrt{\pi a}$ and effective stress ranges $\Delta S_{eff}=S_{max}-S_{cl}$.

Through derivation procedures, McClung (1991) indicates the size of the reversed plastic zone taking fatigue crack closure into account as follows

$$\Delta r_p=\frac{1}{\pi}\left(\frac{U\Delta K}{2\sigma_y-(1-U)\Delta S}\right)^2 \quad (4)$$

where U is the effective stress range ratio, in this case equal to

$$U=\frac{S_{max}-S_{cl}}{S_{max}-S_{min}} \quad (5)$$

The ratio of reversed to forward zone sizes is derived as the form

$$\frac{\Delta r_p}{r_p}=\left(\frac{U/(1-R)}{2-(1-U)(1-R)S_{max}/\sigma_y}\right)^2 \quad (6)$$

where R is the applied stress ratio.

Figure 2 shows the reversed plastic zone size calculated by the ΔK under apparent loading and the one by ΔK_{eff}, according to Eq. (1), which

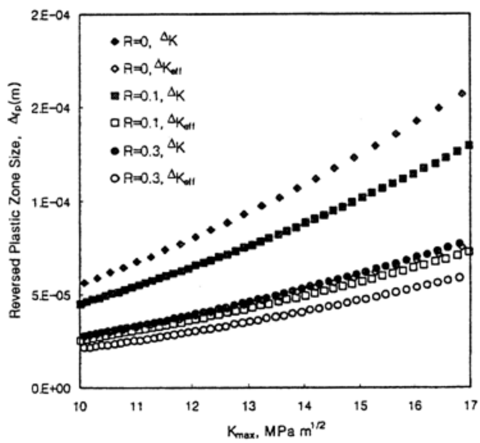


Fig. 2 The reversed plastic zone sizes calculated by ΔK and ΔK_{eff}

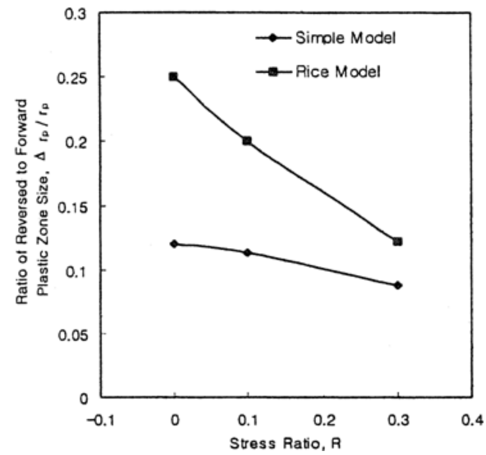


Fig. 3 Ratio of reversed to forward plastic zone size for various stress ratio

includes the opening stress intensity factor K_{op} obtained by Kim(1991)'s experiment along the K_{max}. The reversed plastic zone sizes calculated by ΔK indicate higher than those by ΔK_{eff} for the range of R from 0.0 to 0.3. The range band of the result of ΔK_{eff} is narrower than the one of ΔK, which is similar to the case of crack growth rate.

The ratio of the reversed plastic zone size to the monotonic one is calculated by Eq. (6) and the ratio was postulated by Rice model which didn't consider crack opening behavior for R=0.0, 0.1 and 0.3 are shown in Fig. 3.

The theoretical model by Rice for R=0.0 is 0. 25 as shown in Eq. (2) and for increasing R the value of the ratio of the reversed plastic zone size to the monotonic one deceases apparently.

The results of simple model that includes the K_{op} obtained by Kim(1993)'s experiment for evaluating the plastic zone size show lower than the results of Rice model and decreasing slope along K_{max} is gentler than Rice model. From above results the ratio of the reversed plastic zone size to the monotonic plastic zone size considering K_{op} is much lower than that is calculated by ΔK under applied loading. Dependence upon R about these values of simple model is weaker than Rice model. This is consistent with the result of Fig. 1.

3.2 Mesh generation for reversed plastic zone

The sizes of element along the crack growth line vary proportioning the reversed plastic zone size calculated by Eq. (4). It can be indicated as follows

$$\Delta a \propto \Delta r_p = \frac{1}{\pi}\left(\frac{U\Delta K}{2\sigma_y-(1-U)\Delta S}\right)^2 \quad (7)$$

which Δa is the size of mesh along crack growth line. Δa increases because ΔK becomes larger as crack advances. Therefore, if U is constant through crack growth, the ratio of the initial mesh size, Δa_{ini}, to the final mesh size, Δa_{fin}, is shown as follow

$$\frac{\Delta a_{ini}}{\Delta a_{fin}} = \frac{a_{ini}}{a_{fin}} \quad (8)$$

where a_{ini} and a_{fin} are overall initial and final crack length, respectively. The mesh along crack growth line in Fig. 1 is generated by Eq. (8).

3.3 Analysis of crack opening behavior using plastic zone size

According to Park et al. (1997)'s study the most appropriate mesh size in terms of the monotonic plastic zone size($\Delta a/r_p$) ranges from 0.19 to 0.11 . Also we can easily obtain the ratio ranges from 0.77 to 0.91 of the most appropriate mesh size in terms of the theoretical reversed plastic zone size($\Delta a/\Delta r_p$) by multiplying $4/(1-R)^2$. But these values have approximately 20%

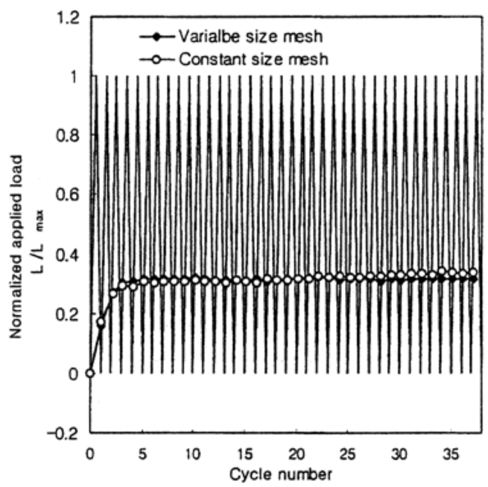


Fig. 4 Behavior of crack opening for constant and variable element sizes of mesh as a function of the applied cycle number (L and L_{max} denote the applied load level and the maximum applied load level, respectively)

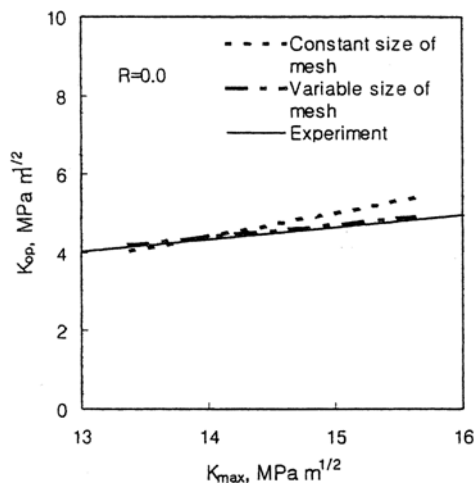


Fig. 5 Comparison of K_{op} among experimental results, constant and variable element sizes of mesh

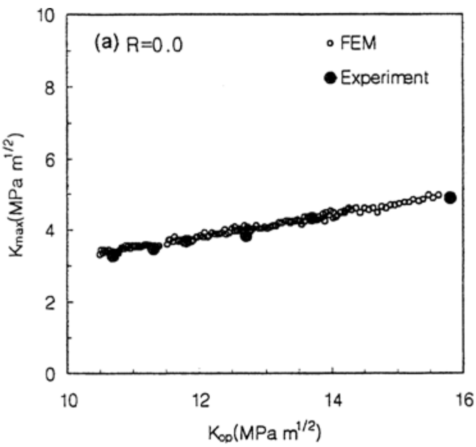
range band, which does not show the independence upon stress ratio R.

Most of researches show that crack growth rate presents unique values when it is related with ΔK_{eff}. It can be predicted that there is the possibility for the reversed plastic zone size to have the unique value when it is calculated by ΔK_{eff} instead of ΔK. In this study the prediction of crack opening level is carried out using the mesh size which is proportional to the reversed plastic zone size considering K_{op} obtained by Kim (1993)'s experiment. The mesh size of crack tip region on crack growth line is determined by the reversed plastic zone size corrected by unique value which is obtained from the reverse of the ratio of the distance from crack tip to remote gauss point in the element along the direction of crack growth.

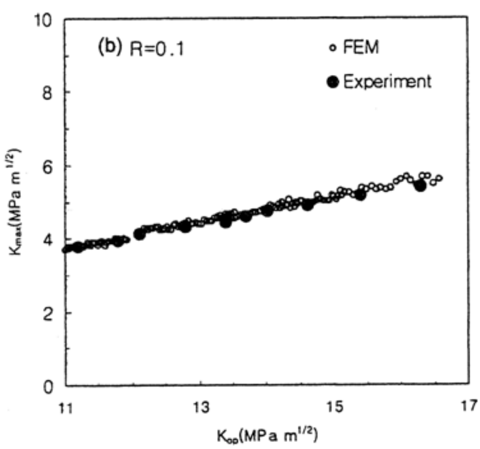
Figure 4 shows the results of the mesh size proportioning the reversed plastic zone size obtained from simple method and the constant mesh size vs. cycle number of crack growth in finite element analysis, where the variable fine mesh sizes change from Δa_{ini}=0.0602mm to Δa_{fin}=0.0847mm and the constant mesh size is 0.07mm. For the constant mesh size the crack opening level increases linearly from initial crack growth. But the crack opening level for variable mesh size is almost flat after crack tip has passed the monotonic plastic zone. There are some differences of opening level at initial and final crack length between the variable and constant mesh size. These results are consistent with other research results that the larger is mesh size of crack tip, the lower the crack opening level.

In Fig. 5, the stress intensity factors of crack opening values after crack tip gets out of the monotonic plastic zone are plotted as a function of the maximum stress intensity factor K_{max} for constant and variable mesh size. The value of K_{max} is varied by changing the crack length under an applied stress of 32 MPa. In the figure the experimental data obtained in Kim (1993)'s work are also shown by the solid line. In case that a single constant mesh size is employed for all crack lengths, the numerical result agrees with experimental one only at a certain value of K_{max}. But the results of using variable mesh size agree with experimental results along entire values of K_{max}. The above results show the same trends of Park et al. (1997)'s ones, which the prediction of crack opening behavior using the constant mesh size was not consistent with experimental results.

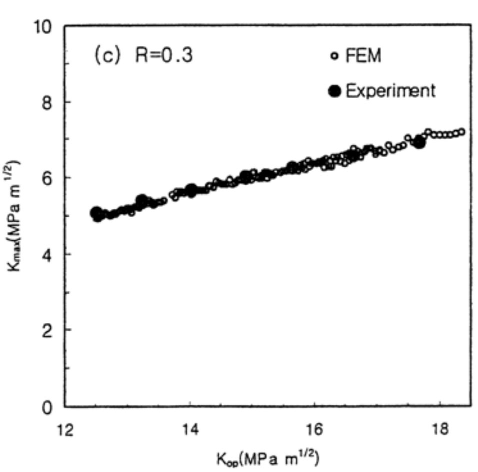
The numerical results obtained from using the variable mesh size proportioning the reversed plastic zone size with the opening stress intensity



(a) R=0.0



(b) R=0.1



(c) R=0.3

Fig. 6 Comparison between experimental and numerical results

factors are compared with the experiment data in Fig. 6. The analysis is performed for R=0, 0.1 and 0.3. For R=0 in Fig. 6(a) the calculations are carried out as dividing into 4 parts, which the variable mesh sizes of the first part is ranging from 0.0362 to 0.044mm, the second part from 0. 0446 to 0.0564mm, the third part from 0.0524 to 0. 0694mm and the fourth part from 0.0602 to 0. 0840mm. The numerical results agree very well with experimental ones through all ranges of the maximum stress intensity factor K_{max}.

For R=0.1 in Fig 6(b). the prediction is performed as dividing into 4 parts, which the variable mesh sizes of the first part is ranging from 0.0362 to 0.0442mm, the second part from 0. 0445 to 0.0566mm, the third part from 0.0525 to 0. 0702mm and the fourth part from 0.0592 to 0. 0838mm. The results agree very well with the same experimental ones as R=0 through all ranges of the maximum stress intensity factor K_{max}.

For R=0.3 in Fig. 6(c) the estimation for crack closure behavior is performed as dividing into 4 parts, which are ranging from 0.0366 to 0. 0440mm, from 0.0449 to 0.0568mm, from 0.0527 to 0.0698mm and from 0.0593 to 0.0838mm. The results agree very well with the same experimental ones as R=0 and R=0.1 through all ranges of the maximum stress intensity factor K_{max}.

The above results indicate that the prediction of crack opening level using the variable mesh size proportioning the reversed plastic zone size with the opening stress intensity factors presents a good agreement with the experiment data regardless of stress ratios

Without the data of crack opening level from experiment the crack opening prediction method using the finite element method and the reverse plastic zone size obtained from simple method is now being investigated.

4. Conclusions

The fatigue cracks opening behavior is investigated using an elastic-plastic finite element analysis. The prediction of crack opening level using the variable mesh size proportioning the reversed plastic zone size is performed and compared with experimentally measured ones. The conclusions obtained are summarized as follows:

(1) The reversed plastic zone sizes calculated by the ΔK indicate higher than those by ΔK_{eff} for the range of R from 0.0 to 0.3. The range band of the result of ΔK_{eff} is narrower than the one of ΔK, which is similar to the case of crack growth rate.

(2) The ratio of the reversed plastic zone size to the monotonic plastic zone size considering K_{op} is much lower than that calculated by ΔK under applied loading. Dependence upon R about these values of simple model is less than Rice model.

(3) For the constant mesh size the crack opening level increases linearly from initial crack growth. The crack opening level for variable mesh size is almost flat after crack tip has passed the monotonic plastic zone.

(4) The prediction of crack opening level using the variable mesh size proportioning the reversed plastic zone size with the opening stress intensity factors presents a good agreement with the experiment data regardless stress ratios.

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